National Academy of Opticianry

# Continuing Education Course

Approved by the American Board of Opticianry

# Basic Math Review

**National Academy of Opticiary**
8401 Corporate Drive #605
Landover, MD 20785
800-229-4828 phone
301-577-3880 fax
www.nao.org

National Academy of Opticianry

**PREFACE:**

This continuing education course was prepared under the auspices of the National Academy of Opticianry and is designed to be convenient, cost effective and practical for the Optician.

The skills and knowledge required to practice the profession of Opticianry will continue to change in the future as advances in technology are applied to the eye care specialty. Higher rates of obsolescence will result in an increased tempo of change as well as knowledge to meet these changes. The National Academy of Opticianry recognizes the need to provide a Continuing Education Program for all Opticians. This course has been developed as a part of the overall program to enable Opticians to develop and improve their technical knowledge and skills in their chosen profession.

The National Academy of Opticianry

**INSTRUCTIONS:**

Read and study the material. After you feel that you understand the material thoroughly take the test following the instructions given at the beginning of the test. Upon completion of the test, mail the answer sheet to the National Academy of Opticianry, 8401 Corporate Drive, Suite 605, Landover, Maryland 20785 or fax it to 301-577-3880. Be sure you complete the evaluation form on the answer sheet. Please allow two weeks for the grading and a reply.

**CREDITS:**

The American Board of Opticianry has approved this course for One (1) Continuing Education Credit toward certification renewal. To earn this credit, you must achieve a grade of 80% or higher on the test. The Academy will notify all test takers of their score and mail the credit certificate to those who pass. You must mail the appropriate section of the credit certificate to the ABO and/or your state licensing board to renew your certification/licensure. One portion is to be retained for your records.

**AUTHOR:**

Diane F. Drake, LDO, ABOM, FCLSA, FNAO

**INTENDED AUDIENCE:**

This course is intended for opticians of all levels

## COURSE DESCRIPTION:

This section will review basic math principles that will enable the optician to use the necessary formulas that are needed on a regular basis to analyze a prescription and fabricate eyewear.

## LEARNING OBJECTIVES / OUTCOME

At the completion of this subsection, the student should be able to:

- Solve problems in addition, subtraction, multiplication, and division of fractions and decimals
- Convert metric values to the US System of Weights and Measures
- Solve problems in basic algebra

National Academy of Opticianry
8401 Corporate Drive #605
Landover, MD  20785
800-229-4828 phone
301-577-3880 fax
www.nao.org

# Basic Math Review

Diane F. Drake, LDO, ABOM, FCLSA, FNAO

Ophthalmic Optics is fairly math intensive. Studies will include a number of formulas as well as the ability to convert forms from fractions to decimals and convert to the metric system. This section will include a very basic review of math.

***Converting Fractions to Decimals***

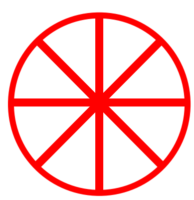
This section will begin with fractions. Fractions can be both common fractions and decimal fractions. For example, 2/5 is a common fraction, and 1.1/2.2 is a decimal fraction. The number above the dividing line in a fraction is the numerator. The number below the dividing line in a fraction is the denominator. A common fraction or decimal fraction may be converted to a decimal by dividing the numerator by the denominator. The answer is the quotient.

Example

1/2 = 0.5

*Fractions*

In introducing algebraic formulas, first of all, we need to understand fractions. An example of a fraction can be illustrated by cutting a pie into 8 equal parts.



Each piece of the pie equals 1/8$^{th}$ of the pie. In the fraction, the top number (1) is the numerator, and the bottom number (8) is the denominator. To convert 1/8$^{th}$ to a decimal, simply divide 1 by 8.

- 1 ÷ 8 = 0.125
- The numerator or top number is 1 and the denominator or bottom number is 8 and the quotient or answer/result is 0.125

As you can see a fraction can also indicate that one number is to be divided by the other.

Example 8/4

- 8 ÷ 4 = 2
- The numerator or top number is 8 and the denominator or bottom number is 4 and the quotient or answer/result is 2

We can also use fractions to compare two things or groups of things.

Example

- 1 inch = $1/12^{th}$ of a foot
- 1 foot = $1/3^{rd}$ of a yard

Algebra is a math form that you will be using many times, every day. Algebra is a method of doing arithmetic/math by means of letters which represent numbers and signs which represent their relation.

Algebra becomes a mathematical language to facilitate the solution of problems in basic optics.

***Algebraic Addition***

Algebraic addition is very simply combining two or more numbers together. If you always think of algebraic addition in terms of dollars and cents, you probably won't make any mistakes. It's really amazing that people who are terrible at math always seem to know their bank balance or how much change they should get back from a purchase. Throughout this section, the examples will be explained mathematically and on occasion, monetarily.

*Algebraic Addition of Two Plus (+) Numbers*

Algebraic addition of two plus (+) numbers will always give an answer that is plus. You simply add the numbers together and give the answer a plus (+) sign.

Example

$$\begin{array}{l} +2.50 \\ \underline{+1.75} \\ +4.25 \end{array} \qquad \text{or } +2.50 + 1.75 = +4.25$$

Monetarily speaking, if you have (+) \$2.50 and you are given (+) \$1.75, how much do you have all told? \$4.25 to the good, or plus.

Example

$$\begin{array}{l} +12.00 \\ \underline{+\ 5.75} \\ +17.75 \end{array} \qquad \text{or } +12.00 +5.75 = +17.75$$

Let's look at this in monetary terms. If you have (+) \$12.00 in your bank account and you receive (+) a check for \$5.75, your new balance would be \$17.75 to the good, or plus.

*Algebraic Addition of Two Minus (-) Numbers*

Algebraic addition of two minus numbers will always give an answer that is minus. You ADD the two numbers together and give the answer a minus (-) sign.

Example

-2.50
<u>-1.75</u> or -2.50 -1.75 = -4.25
-4.25

Monetarily, let us say that you owe (-) a friend $2.50 and you borrow (-) another $1.75. You then owe him (-) $4.25.

Example

-12.50
<u>- 9.50</u> or –12.5 –9.5 = -22.00
-22.00

Let us say that in the first hand of a poker game you lose (-) $12.50 and in the second hand you lose (-) $9.50. You are then $22.00 in the hole (-), and you'd better start playing better before you lose your shirt.

*Algebraic Addition of a Plus and a Minus Number*

Combining a plus and a minus number together will give the same sign in the answer as the sign of the larger number. You take the difference between the two numbers by subtracting the smaller number from the larger.

Example

+12.50
<u>- 9.50</u> or +12.50 -9.50 = +3.00
+ 3.00

In monetary terms, if you have (+) $1 2.50 in your bank account and you write a check to pay a bill (-) for $9.50, you will still have (+) $3.00 in your account.

Example

-9.00
<u>+7.50</u> or –9.00 + 7.50 = -1.50
-1.50

Let us suppose that in the first hand of our poker game you lose (-) $9.00 and in the second hand you win (+) $7.50. Your standing after two hands is then $1.50 in the hole or minus.

*Adding Fractions with Like Denominators*

When adding fractions with like denominators, simply add the numerators (the top number) and write the result over the denominator.

Example

$$\begin{array}{r} 3/5 \\ +\ 1/5 \\ \hline 4/5 \end{array}$$

*Adding Fractions with Unlike Denominators*

In adding fractions with unlike denominators, find the least (sometimes referred to as lowest) common denominator. That is the lowest number that both denominators can be divided into.

Example

$$\begin{array}{r} 3/4 \\ +\ 1/5 \\ \hline \end{array}$$

The lowest number that both 4 and 5 will go into is 20 so the least common denominator in this example is 20. Therefore:

- 20 ÷ 4 = 5
- Then multiply the numerator by the resultant answer
- 3 X 5 = 15 so the new fraction becomes 15/20
- 20 ÷ 5 = 4
- Then multiply the numerator by the resultant answer
- 1x 4 = 20 so the new fraction becomes 4/20

- Now the addition problem becomes

$$\begin{array}{r} 15/20 \\ +\ \ 4/20 \\ \hline 19/20 \end{array}$$

Sometimes the easiest way to find the least common denominator is to simply multiply the two denominators together. This may not result in the *least* common denominator but will work when only two fractions are used. But it will not always work particularly in the case of 3 or more fractions.

*Adding Fractions*

Sometimes in adding fractions, the result is more than a fraction in which case it is necessary to convert to a mixed number.

$$\begin{array}{l} \phantom{+}\ 3/4 \\ \underline{+\ 3/4} \\ \phantom{+}\ 6/4 = 1\ 2/4 = 1\ 1/2 \end{array}$$

The final answer should be reduced to the least common denominator, which simply means that in the case of 1 2/4, 2 will divide into both the numerator of 2 and the denominator of 4 evenly, so divide both numerator and denominator by 2 and your answer is 1/2 making the answer 1 1/2.

*Adding Fractions and Whole Numbers*

Simply annex (place by the side) the fraction to the whole number.

Example

- 5 + 3/4 = 5 ¾

*Adding a Mixed Number and a Whole Number*

Simply add the whole numbers; then annex the fraction to the result.

- 5 + 3 2/3 = 8 2/3

Adding a mixed number with a fraction with a different denominator is easy, simply find the least common denominator as demonstrated above and convert the problem

$$\begin{array}{rcr} 4\ 1/6 & = & 4\ 2/12 \\ \underline{+\quad 3/4} & = & \underline{+\quad 9/12} \\ & & 4\ 11/12 \end{array}$$

*Adding Mixed Numbers*

Find the common denominator, add the whole number and the fractions and reduce the answer if necessary.

$$\begin{array}{rcl} 5\ 2/5 & = & 5\ 4/10 \\ \underline{+\ 3\ 7/10} & = & \underline{+\ 3\ 7/10} \\ & & 8\ 11/10 = 9\ 1/10 \end{array}$$

*Subtracting Fractions*

*Subtracting Fractions with Like Denominators*

Simply subtract the numerators (the top number), and write the result over the denominator.

Example

$$\begin{array}{r} 3/5 \\ \underline{-\ \ 1/5} \\ 2/5 \end{array}$$

*Subtracting Fractions with Unlike Denominators*

In subtracting fractions with unlike denominators, find the least common denominator. That is the lowest number that both denominators can be divided into.

Example

$$\begin{array}{r} 3/4 \\ \underline{-\ \ 1/5} \end{array}$$

The lowest number that both 4 and 5 will go into is 20, so the least common denominator in this example is 20.

- 20 ÷ 4 = 5
- Then multiply the numerator by the resultant answer
- 3 x 5 = 15 so the new fraction becomes 15/20
- 20 ÷ 5 = 4
- Then multiply the numerator by the resultant answer
- 1x 4 = 20 so the new fraction becomes 4/20

Now the subtraction problem becomes:

$$\begin{array}{r} 15/20 \\ \underline{-\ \ \ 4/20} \\ 11/20 \end{array}$$

As mentioned when discussing addition, sometimes the easiest way to find the least common denominator is to simply multiply the two denominators together. This may not result in the least common denominator but will work when only two fractions are used. But it will not always work particularly in the case of 3 or more fractions.

*Subtracting Fractions from Whole Numbers*

Convert the whole number to a fraction. To do this, look at the denominator of the fraction, and convert using the same value.

Example

$$\begin{array}{r} 6 \\ -\ \underline{2\ 3/4} \end{array}$$

Since the fraction includes 4 as the denominator, convert part of the whole number to 4/4. So 6 is converted to 5 4/4.

Becomes:

$$\begin{array}{r} 5\ 4/4 \\ \underline{-\ \ 2\ 3/4} \\ 3\ 1/4 \end{array}$$

To subtract a whole number from a mixed number, find the difference between the whole numbers and annex the fraction.

$$\begin{array}{l} \ 8\ 3/4 \\ \underline{-3\qquad} \\ \ 5\ 3/4 \end{array}$$

To subtract a fraction or a mixed number from a mixed number, find the least common denominator as demonstrated above, subtract the fractions, and finally subtract the whole numbers.

$$\begin{array}{rcr} 4\ 3/4 & = & 4\ 9/12 \\ \underline{-\quad 1/6} & = & \underline{-\ \ 2/12} \\ & & 4\ 7/12 \end{array}$$

*Subtracting Mixed Numbers*

Find the least common denominator, add the whole number and the fractions and reduce the answer if necessary.

$$\begin{array}{rcrcr} 5\ 2/5 & = & 5\ 4/10 & = & 4\ 14/10 \\ \underline{-\ 3\ 7/10} & = & \underline{-\ \ 3\ 7/10} & = & \underline{3\ \ \ 7/10} \\ & & & & 1\ \ \ 7/10 \end{array}$$

In this example, 7/10 could not be subtracted from 4/10, so you had to borrow 10/10 from the 5 making it 4 and adding the 10/10 to the 4/10 making it 14/10…then you could perform your subtraction.

***Multiplication of Fractions***

*Multiplying a Fraction by a Whole Number*

Example

- 2/3 X 5

Convert the whole number into a fraction by simply writing it over the number 1.

Example

- 2/3 X 5/1

Then multiply the numerators. Next the denominators. Finally, if the resultant numerator is larger than the denominator, divide the denominator into it. (Reduce it)

- 2 x 5 = 10 = numerator
- 3 x 1 = 3 = denominator
- 10/3 reduced 10 ÷ 3 = 3 1/3 = quotient

*Multiplying Fractions with Unlike Denominators*

To do this, simply multiply across, or horizontally.

Example

- 2/5 x 1/3 =
- 2 x 1 = 2 numerator
- 5 x 3 = 15 denominator
- 2/5 x 1/3 = 2/15 quotient =answer = result

*Multiplying Two Mixed Numbers*

Convert the mixed number to a full fraction.

- 1 9/16 x 3 3/5
- 1 9/16
  - The full number 1 becomes 16/16
  - 16/16 + 9/16 = 25/16
- 3 3/5
  - The full number 3 becomes 15/5
  - 1 5/5 + 3/5 = 18/5

Now multiply the numerators and denominators

25 x 18 = 450 = numerator
16 x 5 = 80 = denominator
450/80

Divide both numerator and denominator by the lowest common factor which is 10

Becomes

450 ÷ 10 = 45
80 ÷ 10 = 8

Fraction becomes 45/8 or reduced to mixed fraction 5 5/8 = quotient

*Multiplying a Mixed Number by a Whole Number*

First multiply the whole numbers, then multiply the fraction by the whole number and add them together.

Example

- 13 x 3 1/5
- 13 x 3 = 39
- 13 x 1/5 = 13/1 x 1/5 = 13/5 = 2 3/5
- 39 + 2 3/5 = 41 3/5

***Division of Fractions***

To divide a whole number by a fraction, convert the whole number to a fraction by placing it over 1, invert the divisor (the number following the division symbol) and multiply.

Example

- 5 ÷ 1/4 = 5/1 x 4/1 = 20/1 = 20

- 7 ÷ 3/16 = 7/1 x 16/3 = 112/3 = 37 1/3

To divide fractions with unlike denominators, again invert the divisor and multiply.

Example

- 3/4 ÷ 5/7 = 3/4 x 7/5 = 21/20 = 1 1/20

To find the lowest common factor, find the lowest number that will evenly go into each number.

Example

- 15/16 ÷ 5/6 = 15/16 x 6/5 =
- 15/16 x 6/5 = 3/8 x 3/1 = 9/8 = 1 1/8
- 90/80 = 90 ÷ 10 = 9, 80 ÷ 10 = 8 = 9/8 = 1 1/8

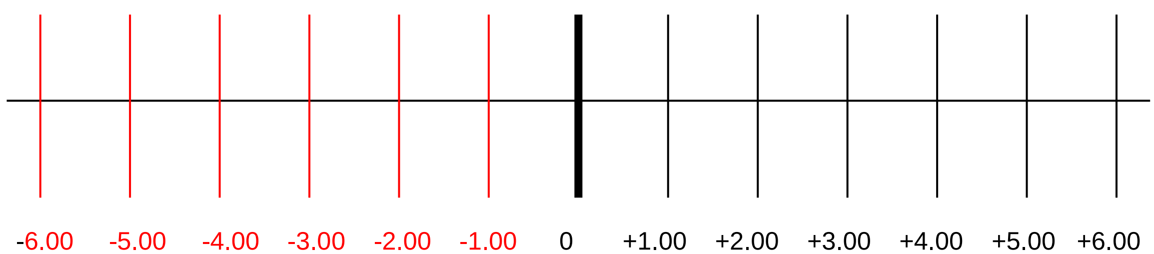
To divide a whole number by a mixed number, invert the divisor and multiply after converting the mixed number into a fraction.

Example

- 15 ÷ 5 3/4 = 15/1 ÷ 23/4 = 15/1 x 4/23 = 60/23 = 2 14/23

***Addition of Signed Numbers***

Using a number line, we see a numbered line containing both positive and negative numbers. This is similar to what you will see on a lensometer, vertometer or lens meter.




A simple example could be: If we want to add +3.00 to -4.00, we start at -4.00 and move 3 places to the right. We would stop at -1.00.

Now let’s go on to the exact directions of adding signed numbers.

*Adding Numbers with Like Signs*

When adding numbers with like signs, simply add the numbers and use the same sign.

Example

- (+2.00) + (+2.00) = +4.00

- Since both numbers are positive, simply add 2.00 plus 2.00 and retain the + sign

Example

- (-2.00) + (-2.00) = -4.00

- Since both numbers are negative, simply add 2.00 plus 2.00 and retain the - sign

*Samples of Like Sign Addition*

- (+1.00) + (+1.00) = +2.00
- (+2.00) + (+1.25) = +3.25
- (-1.00) + (-1.25) = -2.25
- (-4.00) + (-2.50) = -6.50

*Adding Numbers with Unlike Signs*

When adding numbers with unlike signs subtract the smaller number from the larger number and give the result the sign of the larger number.

Example

- (+4.00) + (-1.00) = +3.00

- Since one number is plus (+4.00) and the other is negative (-1.00), simply subtract the 1.00 from the 4.00 and take the sign of the larger which is +, resulting in +3.00

*Samples of Unlike Sign Addition*

- (+1.00) + (-1.00) = 0
- (-2.00) + (+1.25) = -0.75
- (+1.00) + (-1.25) = -0.25
- (-4.00) + (+2.50) = -1.50

***Subtraction of Signed Numbers***

If adding unlike numbers involves subtracting, then subtracting unlike numbers involves adding. Change the sign of the subtracted number and add the two numbers. Using a number line again, this time let's subtract (-3.00) from (+4.00). Simply put, what number added to -3.00 will result in +4.00? In other words, how far and in what direction would you have to advance to arrive at +4.00? The answer is +7.00.

*Samples of Subtraction of Signed Numbers*

- (+1.00) - (-1.00) = +2.00
- (-2.00) - (+1.25) = -3.25
- (+1.00) - (-1.25) = +2.25
- (-4.00) - (+2.50) = -6.50

Subtracting like numbers involve changing the sign of the subtracted number and algebraically adding (meaning subtract the numbers) the two numbers.

Example

- (-4.00) – (-3.00) = (-4.00) + (-3.00) = -1.00
- In other words, what number added to -3.00 will result in -4.00?
- In other words, how far and in what direction would you have to advance to arrive at -4.00?
- The answer is -1.00.

*Samples of Subtraction of Signed Numbers*

- (+1.00) - (+2.00) = -1.00
- (-2.00) - (-1.25) = +0.75
- (+1.00) - (+1.25) = -0.25
- (+4.00) - (+2.50) = -1.50

***Multiplication of Signed Numbers***

When multiplying numbers with like signs, the result will be positive.

Example

- -2.00 X -2.00 = +4.00
- This can also be written (-2.00)(-2.00) = +4.00

- -1.50 X -1.50 - +2.25
- This can also be written (-1.50)(-1.50) = +2.25

*Samples of Like Sign Multiplication*

- (+1.00)(+1.00) = +1.00
- (+2.00)(+1.25) = +2.50
- (-1.00)(-1.25) = +1.25
- (-4.00)(-2.50) = +10.00

When multiplying numbers with unlike signs, the result will be negative.

Example

- +2.00 X -2.00 = -4.00
- This can also be written (+2.00)(-2.00) = -4.00

- +1.50 X -1.50 - -2.25
- This can also be written (+1.50)(-1.50) = -2.25

*Samples of Unlike Sign Multiplication*

- (+1.00)(-1.00) = +1.00
- (+2.00)(-1.25) = -2.50
- (-1.00)(+1.25) = -1.25
- (-4.00)(+2.50) = -10.00

The basic rule for multiplication of signed numbers can be stated as follows

- The product of numbers of like signs is positive, and the product of numbers of unlike signs is negative

***Division of Signed Numbers***

When dividing numbers with like signs, the result will be positive.

Example

- -2.00 ÷ -2.00 = +1.00
- This can also be written (-2.00)/(-2.00) = +1.00

- -1.50 ÷ -1.50 - +1.00
- This can also be written (-1.50)/(-1.50) = +1.00

*Samples of Like Sign Division*

- (+1.00)/(+1.00) = +1.00
- (+2.00)/(+1.25) = +1.60
- (-1.00)/(-1.25) = +0.80
- (-4.00)/(-2.50) = +1.60

When dividing numbers with unlike signs, the result will be negative.

Example

- +2.00 ÷ -2.00 = -1.00
- This can also be written (+2.00)/(-2.00) = -1.00

- +1.50 ÷ -1.50 - -1.00
- This can also be written (+1.50)/(-1.50) = -1.00

*Samples of Unlike Sign Division*

- (+1.00)/(-1.00) = -1.00
- (+2.00)/(-1.25) = -1.60
- (-1.00)/(+1.25) = -0.80
- (-4.00)/(+2.50) = -1.60

The basic rule for division of signed numbers can be stated as follows.

The quotients of numbers of like signs are positive, and the quotients of numbers of unlike signs are negative.
*Rounding OFF*

In rounding off numbers, don't round off until you have arrived at the final answer.

***Multiplication and Division of Decimals***

A decimal number is just a whole number and a fraction written together in decimal form. Any multiplication or division by 10, 100, 1000, etc., simply moves the decimal place to the left or right. For example, multiplying a decimal by 10 would move the decimal point 1 place to the right, or dividing by 100 (10 x 10) would move the decimal point 2 places to the left.

*Multiplication of Decimals*

Decimals are multiplied exactly like whole numbers and then the decimal point is added. For example, you would multiply 25 x 25 in this way.

| | | |
|---|---|---|
| 25 | and 2.5 X 2.5 | 2.5 |
| x 25 | | x 2.5 |
| 125 | | 125 |
| +500 | | +500 |
| 625 | | 6.25 |

The number of decimal places in a decimal is the number of digits (numbers) to the right of the decimal point. The number of decimal places in the product (answer) of a multiplication is the sum total of the decimal places in the numbers that were multiplied. For example, there are 2 decimal places in 140.21 and 3 in 14.021; therefore, there would be 5 decimal places in the product of those two numbers if they were multiplied.

Zeros written to the right of a decimal point with no number other than zero to their right may be dropped in most multiplications. Thus, 4.20 may be written 4.2, but zeros to the right of the decimal point with numbers other than zero to their right cannot be dropped, without changing the value of the number. Thus, you cannot drop the zero in the number 6.105.

Zeros to the left of a decimal point with no number other than zero to their left are added only to make it very clear that the number is a decimal and may be dropped when you multiply. Thus, 0.4 is exactly the same as .4 and 0.23 is the same as .23.

Example

4.32 x 2.10 =

```
   4.32
 x 2.10
   4320
+86400
 9.0720
```

Notice that the 4 decimal places in the answer are counted from right to left.
Answer: 9.072

Example

1600.0 x 0.04 =

```
1600.0
 x 0.04
 64.000
```

Answer: 64.0 or 64

*Division of Decimals*

Divisions may be written in the form a/b = c, where a is the dividend, b is the divisor, and c is the quotient. As with multiplication, you divide decimals exactly like you do whole numbers and then you find the decimal place. For example, dividing 126 by 6 gives 21 as an answer

$$6 \overline{)126} \quad \text{quotient } 21$$

and dividing 12.6 by 6 gives the answer 2.1.

$$6 \overline{)12.6} \quad \text{quotient } 2.1$$

Notice that the decimal point in the quotient (answer) is directly above the decimal point in the dividend.

Anytime there is a space or a number of spaces between the decimal point and the first number of the quotient, you must add zeros to complete your answer.

For example, if you were to divide 8 into .248, you must add a zero after the decimal point but before the three in the quotient (answer). As a result, the problem and correct answer would look like this:

$$8 \overline{).248} \quad \text{quotient } .031$$

To carry out a division as far as necessary you must often add zeros to the dividend. As we have seen before, this does not change the value of the number

Thus, in .9/2 = .45 it is necessary to add a zero to produce

$$2\overline{\smash{)}.90}\quad = .45$$

When the divisor is a decimal, you may change it to a whole number by moving the decimal point to the right. When the decimal point is moved in the divisor, it must be moved the same number of places in the dividend. If you move the decimal two places in the divisor, you must also move it two places in the dividend. For example

**0.88 = .2 $\overline{\smash{)}0\,.88}$ = 2$\overline{\smash{)}8.8}$ = 4.4**

We must move the decimal in both divisor and dividend one place to the right. This then gives us 4.4 as a result. When the divisor is a whole number, we just divide like whole numbers.

***Algebraic Expressions***

In addition to the explicit numbers, 0-1-2-3, etc., used in arithmetic, in algebra we use letters to represent numbers. When a letter is used in this manner, it is called a literal number.

The signs, +, -, x and / for the basic operations of add, subtract, multiply and divide are used with literal numbers to create a mathematical statement called an equation.

A formula (equation) is a mathematical statement that two quantities are equal.

For example

$$D = \frac{1}{7}$$

The literal number D represents dioptric value. The literal number f represents focal length.

In the remainder of the text, a reciprocal formula such as $\frac{1}{f}$ will be written as 1/f.

An algebraic expression is a set of variables with assigned values.

The use of x and y are common but then again so are any number of letters.

Example

- Find the value of $2x + 3y$, when $x = 8$ and $y = 5$

    - Remember that the variable x or y is tied to the numerical value so you will multiply them in the equation.
    - $2(8) + 3(5) = 2 \times 8 + 3 \times 5 =$
    - $16 + 15 = 31$

Here's an example of an equation in which a fraction is expressed.

Find the value of $(a + c)\div(a - c)$ when $a = 6$ and $c = 3$

$$\frac{a + c}{a - c} = \frac{6 + 3}{6 - 3} = \frac{9}{3} = 3$$

In this example, we simply substituted the numerical value for the letter in the numerator and the denominator and solved the resulting fraction. This process is called "simplifying".

This example demonstrates an expression in parenthesis. Perform the task in the parenthesis first.

Find the value of $6(l + 1)$ when $l = 10$ and $w = 8$.

- First of all simplify by changing the letters to the corresponding numbers…the equation becomes:

- $6(10 + 8) = 6(18)$ or $6 \times 18 = 108$

An exponent is a number which represents a "power". It is placed to the right and slightly above a letter or number.

- $x^2 + y^2 =$ when $x = 3$ and $y = 5$
    - The number $^2$ at the right means squared, which also means the number multiplied by itself
    - 3 squared = $3 \times 3 = 9$
    - 5 squared = $5 \times 5 = 25$
    - $9 = 25 = 34$

*Finding the Value of an Expression when We Have the Value of One Expression and a Numerical Value*

Example

- Find the value of F when C = 20
- Formula is:
  - F = 1.8C + 32
  - F = 1.8(20) = 32
  - F = 36 + 32
  - F = 68

***Solving Equations***

An equation is an algebraic sentence.  Solving an equation simply means to find the unknown value.  Finding the value makes the statement true.

Example

- n + 2 = 8
- What is the value of n
- To solve this one, simply subtract 2 from both sides of the equation
- n + 2 – 2 = 8 – 2
- n = 6

Here's another example but the equation includes subtraction.

Example

- n - 2 = 8
- What is the value of n
- To solve this one, simply add 2 to both sides of the equation
- n - 2 + 2 = 8 + 2
- n = 10

Here is an example of an equation involving multiplication.

Example

- $n^2 = 8$
- What is the value of n
- To solve this one, simply divide both sides of the equation by 2
- $n^2 \div 2 = 8 \div 2$
- $2 \div 2 = 1$
- $8 \div 2 = 4$
- n1 = 4
- n = 4

Here is an example of an equation involving division.

Example

- n/2 = 8
- What is the value of n
- To solve this one, simply multiply both sides of the equation by 2
- n/2 x 2 = 8 x 2
- n = 8 x 2
- n = 16

*Equations Involving a Combination of Terms*

This example combines multiplication and addition.

- 7n + 6 = 41
- Find n
- First of all, subtract 6 from both sides of the equation
- 7n + 6 – 6 = 41 – 6
- 7n = 35
- Then divide both sides of the equation by 7
- $7n \div 7 = 35 \div 7$
- n = 5

***Metric System***

The metric system, developed in the late 1700's was a culmination of years of study to devise a decimal system of measurement. All major countries today, except the United States, have adopted the metric system.

However, any American involved in international trade must know and use metrics and how to convert U.S. units to this system. Opticians as members of an international industry are no exception. The metric system is based on decimals. Changing from one unit to another requires only the movement of the decimal point.

1 meter (m) = 1 meter
100 centimeters (cm) = 1 meter 1000 millimeters (mm) = 1 meter

You can see that the metric system is quite similar to our monetary system of dollars and cents. When you are making your conversions from one unit to another, think in terms of dollars and cents and you probably won't make a mistake.

For illustrations only consider:

- 1 m = 1 dollar
- 1 cm = 1 cent
- 1 mm = 1 mil
    - A mil is 1/1000

If you consider cm as cents, mm as mils and meters as dollars you will find your conversions much easier. Most of us understand that the notation $0.40 means 40 cents, so in a similar fashion 0.40 m equals 40 cm. This process is actually accomplished by moving the decimal place in 0.40 m two places to the right to give us 40.0 cm. Thus, we will find that conversion from one metric unit to another simply means moving the decimal point. This requires deciding (1) how many places to move the decimal point and (2) whether to move it right or left.

Furthermore, considering that everything is based on the meter, will the amount you are looking for be less or more?

*Metric Conversions*

A meter may be divided into parts

- Tenths
- Hundredths
- Thousandths
- Others

For the purpose of this basic course of study, we will only go into thousandths.

- Meter = m
- Decimeter = dm
- Centimeter = cm
- Millimeter = mm

- 1m = 10 dm
- 1m = 100 cm
- 1m = 1000 mm

When converting from meters to decimeters, centimeters or millimeters move the decimal point to the right.

- m = 10 dm
- m = 100 cm
- m = 1000 mm

When converting from millimeters, centimeters or decimeters to meters move the decimal point to the left.

- 1 mm = .001 m
- 1 cm = .01 m
- 1 dm = .1 m

In trying to remember which direction to move a decimal when converting from meters to decimeters, centimeters, or millimeters or the reverse, just remember:

- Would you have more or less of the unit that you desire?
- If the unit that you desire is less, then move the decimal to the right
- If the unit that you desire is more, then move the decimal to the left

For example, if you are given a length in meters and you require the length in centimeters, then you must have more centimeters than you had meters because each centimeter is smaller than each meter. This means that you would move the decimal 2 places to the right. Conversely, if you were converting from centimeters to meters, you would have to move the decimal point to the left 2 places. A meter is a much larger unit of length than a centimeter, thus you would have to have fewer meters than you had centimeters.

| When converting | Move decimal |
|---|---|
| m to dm | 1 place right |
| dm to cm | 2 places right |
| cm to mm | 1 place right |
| mm to m | 3 places left |
| mm to cm | 1 place left |
| mm to dm | 2 places left |
| mm to m | 3 places left |

Example

How many cms are there in 0.40 m?

Solution

- There are more centimeters in meters than meters in centimeters, therefore, move the decimal 2 places to the right.
- The answer is:

- 0.40 m = 0.40. = 40 cm

Example

How many cms are there in 0.20 m?

Solution

- There are more centimeters in meters than meters in centimeters, therefore, move the decimal 2 places to the right.
- The answer is:
- 0.20 m = 0 2 0. = 20.0 cm

Example

Convert 358 cm to mm.

Solution

- There are more millimeters in centimeters than centimeters in millimeters, therefore, move the decimal 1 place to the right.
- The answer is:
- 358 cm = 358.0 = 358 mm

Example

Convert 0.0008 mm to m.

Solution

- There are more millimeters in meters than meters in millimeters, therefore, move the decimal 3 places to the left.
- The answer is:
- 0.0008 mm =. 000000 8 = .0000008 m

Example

How many cms are there in 250 mm?

Solution

- There are more millimeters in centimeters than centimeters in millimeters, therefore, move the decimal 1 place to the left.
- The answer is:
- 250 mm = 2 5.0 = 25.0 cm

Example

You measure a distance to be 0.12 mm. How many cms are there in 0.12 mm?

Solution

- There are more millimeters in centimeters than centimeters in millimeters, therefore, move the decimal1 place to the left.
- The answer is:
- 0.12 mm =. 0 1 2 = 0.012 cm

*English to Metric Conversions*

Another conversion you are likely to make is between inches and metric units. There are approximately 40 inches in 1 meter, actually 39.37 inches; however, we use 40 inches in most formulas.

- 1 yard = 36 inches
- 1 meter = 39.37 inches
    - For approximation purposes only 1 m = 40 inches
- 1 inch = 25.4 mm
- 1 inch = 2.54 cm
- 1 inch = 0.254 dm
- 1 inch = 0.0254 m

When converting from metric to feet, first convert to inches and then divide by 12.
- 1 foot = 12 inches

When converting from metric to yards, first convert to inches and then divide by 36.
- 1 yard = 36 inches

Example

- 80 in. = 80/40 m = 2 m
- 5 in. = 5/40 = 0.125 m

Should you desire a length, given in meters, converted to inches, multiply by 40.

Example

- 0.5 m = 0.5 x 40 = 20 in.
- 4.0m = 4.0 x40 = 160 in.
- 1.25m = 1.25x40 = 50 in.

If you need a length, in inches, converted to centimeters or millimeters, first convert the inches to meters (divide by 40), then convert to the desired unit by moving the decimal place. Conversely, if you wish to convert from cm or mm to inches, first convert to meters by moving the decimal; then multiply by 40 to convert the meters to inches.

Example

How many mm are there in 16 inches?

Solution

- Convert 16 in. to meters by dividing by 40.
- Then move the decimal 3 places to the right.
- 16 in =16 m/40 = O.4 m = O400. mm = 400 mm

Example

How many inches are there in 20 cm?

Solution

- Convert 20 cm to meters by moving the decimal 2 places to the left.
- Then multiply the meters by 40 to obtain inches.
- 20 cm = 0.20 m 0.20 x 40 = 8 inches

How many cms are there in a distance of 60 inches?

Solution

- Divide 60 inches by 40 to obtain meters.
- Then move the decimal 2 places to the right to obtain cm.
- 60 in. = 60/40 = 1.5 m
- 1.5m= 150.= 150cm

## *Metric System Review*

*Prefixes*

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 Meter (M.) | = | 39.37 inches | | | | |
| 0.0254 M. | = | 1 inch | | | | |
| 1 Decameter | = | 10 Meters | = | 393.70 Inches (10xm) | | |
| 1 Hectometer | = | 100 Meters | = | 382.08 feet | = | 0.0621 Mile (100xm) |
| 1 Kilometer | = | 1000 Meters | = | 3,280.8 feet | = | 0.621 Mile (1000xm) |
| 1 Myriameter (10,000xM) | = | 10,000 Meters | = | 32,808.0 feet | = | 6.21 Mile |
| Deci- | = | One-tenth (1/10) [0.1] Meter [M/10] | | | | |
| 1 Meter | = | 10 decimeters (dm) | | | | |
| 0.254 dm | = | 1 Inch | | | | |
| Centi- | = | One-One Hundredth (1/100) [0.01] Meter [m/100] | | | | |
| 1 Meter | = | 100 Centimeters (cm) | | | | |
| 2.54 cm | = | 1 Inch | | | | |
| Milli- | = | One-One Thousandth (1/1000) [0.001] Meter [m/1000] | | | | |
| 1 Meter | = | 1000 Millimeters (mm) | | | | |
| 25.4 mm | = | 1 Inch | | | | |
| 1 mm | = | 0.03837 inch | | | | |

### *Conversion*

Many of our optical formulae require that Millimeters be converted to Meters and Meters to Millimeters.

*Meters to Millimeters*

- Move the decimal (multiply by 1000) place three places to the right
- i.e.: 0.012 m = 12 mm
- 0.012 m x 1000 = 12 mm
- i.e.: 0.006 m = 6 mm
- 0.006 m x 1000 = 6 mm

*Millimeters to Meters*

- Move the decimal (divide by 1000) place three places to the left
- i.e.: 12 m = 0.012 mm
- 12 m /1000 = -.012 mm
- i.e.: 6m= 0.006 mm
- 6 m /1000 = 0.006 mm

*Meters to Centimeters*

- Move the decimal (multiply by 100) place two places to the right
- i.e.: 0.012 m = 1.2 cm
- 0.012 m x 100 = 1.2 cm

*Centimeters to Meters*

- Move the decimal (divide by 100) place two places to the right
- i.e.: 1.2 cm = 0.012 m
- 1.2 cm / 100 = 0.012m

*Centimeters to Inches*

- 2.54 cm = 1 inch
- i.e.: 10 cm = 3.93 inches
- 10 / 2.54 = 3.93 in

*Inches to Centimeters*

- Inches X 2.54 = Centimeters
- i.e.: 3 Inches = 7.62 cm
- 3 x 2.54 = 7.62

Remember the Metric system works on multiples of ten, based on the standard of the Meter.

- Decameter = 10 Meters — 1 Meter = 0.1 Decameter
- Hectometer = 100 Meters — 1 Meter = 0.01 Hectometer
- Kilometer = 1000 Meters — 1 Meter = 0.001 Kilometer

- Meter
- Decimeter = 0.1 Meters (1/10) — 1 Meter = 10 decimeters
- Centimeter = 0.01 Meters (1/100) — 1 Meter = 100 centimeters
- Millimeter = 0.001 Meters (1/1000) — 1 Meter = 1000 millimeters

National Academy of Opticianry
8401 Corporate Drive #605
Landover, MD  20785
800-229-4828 phone
301-577-3880 fax
www.nao.org